Shocks Correlation and Its Effect on an Insurer's Reinsured Proportion of Wealth and Optimal Strategy for Investment under Constant Elasticity of Variance Model and Exponential Utility Preference

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Abstract
In this study, it is assumed that in a financial market the risky asset was governed by the constant elasticity of variance (CEV) model and the surplus process of an insurer approximated by a stochastic differential equation (SDE) where the insurer traded two assets-a risky asset (stock) and a risk free asset (bond). The insurer was permitted to take proportional reinsurance and the effect of the correlation of the Brownian motions investigated when they did not correlate and when they did. Hamilton-Jacobi-Bellman equations (HJB) were derived using the Ito's lemma from which the insurer's optimal investment strategy and the reinsured proportion were obtained. It was observed that the investment strategies differed by the fraction, $\frac{\rho b(p(t)-1)}{\beta}$ *and optimal reinsured proportion by the fraction,* $\frac{\rho\,\beta s^{2\gamma}(t)}{b}$*. Both were found to be horizon dependent, therefore it is recommended that this condition of horizon dependency should be taken into consideration when making investment decision.*

Keywords: *Constant elasticity of variance (CEV) model, Exponential utility preference, Hamilton-Jacobi-Bellman equations (HJB), insurer, shocks correlation.*

1. Introduction

Insurance is one of the social sciences essentially designed for risk taking. This process of risk taking entails the pooling together of resources of many individuals.

Daily, people are exposed to infinite number of risks that may affect their persons or their properties. The insured transfers those risks to an insurer, at a fixed cost called the insurance premium. According to Gu (2014), investment and reinsurance are two main ways for insurers to balance their profits and risks. Reinsurance is a transaction whereby one insurance company agrees to indemnify another insurance company against all or part of the loss that the latter sustains under a policy or policies that it has issued. The company that grants insurance to an insurer is called a reinsurer.

The purpose of reinsurance is to spread risk. This protects insurers against extraordinary and/or unforeseen losses by allowing them to spread their risks. A case of a catastrophic fire in an industrial enterprise which could devastate its insurer financially is an example that requires reinsurance.

A contractual relationship between an insurer and an insured which provides the insured the financial means to compensate a pecuniary claim or reduce the consequence of personal injury sustained in a claim is an example of insurance contract or policy.

A primary insurer, which is the insurance company, an individual or business, purchases a policy from and transfers her risks to a reinsurer through a process called premium payment to reinsurance companies. The premiums primary insurers pay to reinsurers may be reduced by any commission the insurer pays to their reinsurer.
Recently, insurance companies play more active role in the financial markets. They invest in the financial market, purchase reinsurance from the reinsurer and acquire new businesses; acting as reinsurers to other insurers, to avoid risk.

Many authors have contributed to insurance literatures. Wang et al. (2007) applied martingale method to study the optimal portfolio selection for insurer under the mean-variance criterion as well as the expected constant absolute risk aversion (CARA) utility maximization. In addition to the risk of market, the insurer also takes into account the risk of insurance. The risk of insurance cannot be avoided by single investing in the bond and other assets in market.

Bauerle (2005) investigated quota-share reinsurance and investment which were previously looked into by proportional reinsurance that was accessible in which he minimized the expected quadratic distance of the terminal value over a positive constant and solved the related mean-variance problem successfully.

Zhibin and Bayraktar (2014) studied the problem of optimal Reinsurance–investment in a constant elasticity of variance stock market for Jump-diffusion risk model and obtained explicit expressions for the optimal strategies and value function. They used numerical examples to show the impact of model parameters on the optimal strategies.
Yang and Jiaqin (2016) considered optimal investment-consumption-insurance with random parameter. They discussed that optimal investment, consumption, and life insurance purchase problem for a wage earner in a complete market with Brownian information. They assumed that the parameter governing the market model and the wage earner, including the interest rate, appreciation rate, volatility, force of mortality, premium-insurance ratio, income and discount rate, are all stochastic.

Deng et al (2015) in their study of the optimal proportional reinsurance and investment for a constant elasticity of variance model under variance principle assumed that the insurer's surplus process follow a jump-diffusion process.
Osu et al. (2014) studied the survival of insurance company when consumption was involved under power and exponential utility functions and found that the optimal strategies obtained for both utility functions yielded results that are alike.

Ihedioha and Osu (2015) studied the optimal portfolio of an insurer and a reinsurer under proportional reinsurance and power utility preference in which the insurer's and the reinsurer's surplus processes were approximated by Brownian motion with drift.

Li et al. (2015) studied a time – consistent reinsurance – investment strategy for a mean – variance insurer under stochastic interest rate model and inflation risk and derived the time – consistent reinsurance – investment striates as well as the corresponding value function for the mean – variance problem explicitly. They also study the optimal investment problem for an insurer and a reinsurer under the proportional reinsurance model. The insurer's and reinsurer's surplus processes are both approximated by a Brownian motion with drift and the insurer can purchase proportional reinsurance from the reinsurer. Both the insurer and the reinsurer are allowed to invest in a risk-free asset and a risky asset. They first study the optimization problem of minimizing the ruin probability for the insurer. Then according to the optimal reinsurance proportion chosen by the insurer, they study two optimal investment problems for the reinsurer: the problem of maximizing the exponential utility and the problem of minimizing the ruin probability. By solving the corresponding Hamilton-Jacobi-Bellman (HJB) equations, they derived optimal strategies for both the insurer and the reinsurer explicitly. Furthermore, they find that the reinsurer's optimal strategies under the two cases are equivalent for some special parameters. Finally they present numerical simulations to illustrate the effects of the model parameters on the strategies.

Zhou and Cai (2014) studied the optimal dynamic risk control for insurers with state – dependent income in which they investigated optimal forms of dynamic reinsurance policies among a class of general reinsurance strategies. The original surplus process of an insurance portfolio is assumed to follow a Markov jump process with state-dependent income. They assumed that the insurer uses a dynamic reinsurance policy to minimize the probability of absolute ruin, where the traditional ruin can be viewed as a special case of absolute ruin. In terms of approximation theory of stochastic process, the controlled diffusion model with a general reinsurance policy is established strictly. In such a risk model, absolute ruin is said to occur when the drift coefficient of the surplus process turns negative, when the insurer has no profitability any more. Under the expected value premium principle, they rigorously prove that a dynamic excess-of-loss reinsurance is the optimal form of the reinsurance among a class of general reinsurance strategies in a dynamic control framework.

Zhibin and Guo (2010) considered the optimal proportional Reinsurance under two criteria maximizing the expected utility and minimizing the value at risk and proved the existence and uniqueness of the optimal strategies and Pareto optimal solution, and give the relationship between the optimal strategies.

Gu et al. (2010) studied the constant elasticity of variance model for proportional reinsurance and investment strategies in which the claim process is assumed to follow a Brownian motion with drift, while the price process of the risky asset is described by the constant elasticity of variance model and obtain the optimal reinsurance and investment strategies.
The above reviewed works did not consider the case of correlation of Brownian motions. Therefore, in this study, we shall consider the case of a reinsurer assessing the impact of the correlation of the Brownian motions on the insurer's optimal investment strategy and reinsured proportion where the insurer's surplus process is approximated by constant elasticity of variance (CEV) model, and an insurer could purchase proportional reinsurance from the reinsurer.

Ito's lemma shall be used in obtaining the Hamilton–Jacob-Bellman (HJB) equation which will be solved to get the insurer's optimal investment in the risky and the optimal reinsured proportion after which we investigate the effects of the correlation of the Brownian motions on them.

2. The Model Formulation the Model

The constant elasticity of variance (CEV) model is one-dimensional diffusion process that solves a stochastic differential equation (SDE). It is a natural extension of the geometric Brownian motion (GBM). The constant elasticity of variance model was originally proposed by Cox and Ross as an alternative diffusion process for European option pricing. Compared to Geometric Brownian Motion (GBM), we see that the advantages of the constant elasticity of variance (CEV) model are that the volatility rate has correlation with risky asset price and can explain the empirical bias such as volatility smile.

The constant elasticity of variance (CEV) model is giving as;
$dS(t) = S(t)[\mu dt + \beta S(t)^{\gamma} dZ(t)]$,
where μ is a long term rate of return, γ is the elasticity parameter satisfying$\gamma > 0$, $\beta S^{\gamma}(t)$ is the volatility, and $Z^{(2)}(t)$ is a standard Brownian motion .

Remark: when the elasticity parameter γ equals zero, the constant elasticity of variance (CEV) model reduces to Geometric Brownian motion.

2.1. The Model

Suppose the claim process $C(t)$ of an insurance company is described by;

$$dC(t) = adt - bdZ^{(1)} \quad (1)$$

Where a and b are positive constant and$Z^{(1)}(t)$ is a standard Brownian motion defined on a complete probability space $(\Omega, f, (f_t);\ t > 0)$.
Assuming also that the premium rate is

$$c = (1+\theta)a. \quad (2)$$

where $\theta > 0$is the security risk premium.
Applying (2) in (1), the surplus process of the insurer is now given as

$$dR(t) = cdt - dC(t) = a\theta dt + bdZ^{(1)}(t) \quad (3)$$

We assume that the insurance company has the permission to purchase proportional reinsurance to reduce her risk and pays reinsurance premium continuously at the rate of $(1+\eta)ap(t)$where $\eta > \theta > 0$ is the safety loading of the reinsurer.
The surplus of the insurance company is then given as

$$dR(t) = \big(\theta - \eta p(t)\big)adt + b\big(1 - p(t)\big)dZ^{(1)}(t), \quad (4)$$

We also assume that the insurer invests his surplus in a market consisting of two assets; a risky asset (stock) and a riskless asset (bond).Denoting the prices of the risky and riskless assets by$S(t)$ and$B(t)$ respectively and the dynamics of the price of the risky asset is modeled using the constant elasticity of variance (CEV) as

$$dS(t) = S(t)\big[\mu dt + \beta S^{\gamma}(t)dZ^{(2)}(t)\big], \gamma > 0, 0 \le \beta \le 1. \quad (5)$$

where μ denotes the appreciation rate of the risky asset ,$\beta S^{\gamma}(t)$ its volatility and$Z^{(2)}(t)$ another standard Brownian motion defined on a complete probability space.
Let the evolution of the price of the riskless asset be given by the equation

$$dB(t) = kB(t)dt;\ B(0) = 1, \quad (6)$$

where k is a constant.
If $W(t)$is the total amount of money the insurer and has for investment and he invests $\pi(t)$in the riskyasset, then his investments on the riskless asset is$[W(t) - \pi(t)]$. Corresponding to the policy π, the admissible strategy$[\,p(t)\,, \pi(t)]$, the wealth processes of the insurer evolves according to the stochastic differential equations (SDE)

$$dW^{\pi} = \pi(t)\frac{dS(t)}{S(t)} + [W(t) - \pi(t)]\frac{dB(t)}{B(t)} + dR(t). \quad (7)$$

Substituting in (7) the expressions for $\frac{dS(t)}{S(t)}, \frac{dB(t)}{B(t)}$, d$R(t)$, making use of equations (5), (6), and equation (7) becomes

$$dW^{\pi}(t) = \pi(t)\big[\mu dt + \beta S^{\gamma}(t)dZ^{(2)}(t)\big] + [W(t) - \pi(t)]\ kdt + \big(\theta - \eta p(t)\big)adt + b(1 - p(t)dZ^{(1)}(t). \quad (8)$$

Employing the fact that

$$\left.\begin{array}{l}(dt)^2 = (dt)(dZ^{(1)}(t) = (dt)(dZ^{(2)}(t) = 0,\\ \left[dZ^{(1)}(t)\right]^2 = \left[dZ^{(2)}(t)\right]^2 = dt,\\ \left[dZ^{(1)}(t)\right]\left[dZ^{(2)}(t)\right] = \rho dt\end{array}\right\}, \qquad (9)$$

the quadratic variation of the wealth process of the insurer is

$$(dW^{\pi})^2(t) = [\beta^2\pi^2(t)s^{2\gamma)}(t) + b^2\left(1-p(t)\right)^2]dt, \qquad (10)$$

when the shocks do not correlate, and

$$(dW^{\pi})^2(t) = [\beta^2\pi^2(t)s^{2\gamma}(t) + 2\beta b\left(1-p(t)\right)s^{2\gamma}(t)\pi(t)\rho + b^2\left(1-p(t)\right)^2]dt, \quad (11)$$

when the shocks correlate.

Suppose the insurer has exponential utility preference given as

$$U(w) = \frac{-e^{-\phi w}}{\phi}, \quad \phi > 0, \qquad (12)$$

then the investors insurer's problem can then be written as

$$V(T,w) = Max_{\pi}E^{(t,w)}[U(W^{\pi})]. \qquad (13)$$

subject to

$$dW^{\pi}(t) = \pi(t)\left[S(t)[\mu dt + \beta S^{\gamma}(t)dZ^{(1)}(t)]\right] + [W(t)-\pi(t)]\ kdt + \left(\theta - \eta p(t)\right)adt + b(1-p(t)dZ^{(1)}(t).$$

3. The Optimization Programme

The insurer's optimal investment strategy and the proportion reinsured are obtained in this section.

3.1 The Case of None Correlation Shocks

We derive the Hamilton–Jacobi–Bellman (HJB) partial differential equation starting with the Bellman equation

$$V(w,T) = Max_{\pi}E[V(w',T)]. \qquad (14)$$

where w', denotes the wealth of the insurer at time T and equation (14) can be written as:

$$Max_{\pi}E[V(w,t+\Delta t,T)] - V(w,t;T) = 0. \qquad (15)$$

Dividing both side of the equation byΔt and taking the limit as Δt tends to zero, gives the Bellman equation

$$Max_{\pi}\frac{1}{\Delta t}E(dV) = 0. \qquad (16)$$

Applying the Ito's lemma which state that

$$dV = \frac{\partial V}{\partial w}dt + \frac{\partial V}{\partial w}dw + \frac{1}{2}\frac{\partial^2}{\partial w^2}(dw)^2, \qquad (17)$$

and substituting for $dW^{\pi}(t)$and $(dW^{\pi})^2(t)$in (16)using (8) and (10) respectively yields the stochastic differential equation (S D E),

$$dV = \frac{\partial V}{\partial t}dt + \frac{\partial V}{\partial w}\left\{\pi(t)\left[S(t)[\mu dt + \beta S^{\gamma}(t)dZ^{(1)}(t)]\right] + [W(t)-\pi(t)]\ kdt + \left(\theta - \eta p(t)\right)adt + b(1-p(t)\right\} + \frac{\partial^2 V}{\partial w^2}\left\{[\beta^2\pi^2(t)s^{2\gamma}(t) + b^2\left(1-p(t)\right)^2]dt\right\}, \qquad (18)$$

which when substituted in (16) simplified yields the required Hamilton-Jacobi-Bellman (HJB) equation

$$\frac{\partial V}{\partial t} + \frac{\partial V}{\partial w}\{\pi(t)[\mu S(t)] + [W(t)-\pi(t)]\ k + \left(\theta - \eta p(t)\right)a + b(1-p(t)\} + \frac{\partial^2 V}{\partial w^2}\left\{[\beta^2\pi^2(t)s^{2\gamma}(t) + b^2\left(1-p(t)\right)^2]\right\} = 0, \qquad (19)$$

where

$$E\left(dZ^{(1)}(t)\right) = E(dZ^{(2)}(t) = 0\,. \qquad (20)$$

Rewriting (20) in a different, we have

$$V_t + \{(\mu S - K)\pi + kW + \left(\theta - \eta p(t)\right)a + b(1 - p(t))\}V_w + \left\{\beta^2\pi^2(t)s^{2\gamma}(t) + b^2\left(1 - p(t)\right)^2\right\}V_{ww} = 0. \quad (21)$$

The homogeneity of the objective function, the restriction and the terminal condition, lead to the conjecture that the value function V must be linear to $\frac{-e^{-\phi w}}{\phi}$. Therefore, let

$$V(w,t,T) = g(t,T)\left(\frac{-e^{-\phi w}}{\phi}\right), \quad (22)$$

such a function such that at the terminal date T,

$$g(T,T) = 1 \quad (23)$$

Then we obtain from (22)

$$V_t = \frac{-e^{-\phi w}}{\phi}g' \,;\, V_w = e^{-\phi w}g; V_{ww} = -\phi e^{-\phi w}g. \quad (24)$$

Using (24) in (19) and simplifying yields the new HJB equation

$$\frac{-1}{\phi}g' + \left\{\{(\mu S - k)\pi + kW + \left(\theta - \eta p(t)\right)a + b(1 - p(t))\} - \frac{\phi}{2}\left\{\beta^2\pi^2(t)s^{2\gamma}(t) + b^2\left(1 - p(t)\right)^2\right\}\right\}g = 0. \quad (25)$$

To obtain the optimal value $\pi^*(t)$ of $\pi(t)$ we differentiate (25) with respect to $\pi(t)$to obtain

$$(\mu S(t) - k) - \frac{\phi}{2}[2\pi(t)\beta^2 s^{2\gamma}(t)] = 0, \quad (26)$$

which simplifies to

$$(\mu S(t) - k) - \emptyset\beta^2 s^{2\gamma}(t)\pi(t) = 0, \quad (27)$$

and we obtain the optimal strategy as

$$\pi^*(t) = \frac{(\mu S(t) - k)}{\emptyset\beta^2 s^{2\gamma}(t)}. \quad (28)$$

This is the insurer's optimal investment in the risky asset (stock) which, clearly is horizon dependent.

Also differentiating equation (25) with respect to $p(t)$, we obtain

$$-a\eta - b - \frac{\emptyset}{2}b^2(2(1 - p(t)) \times -1\,) = 0, \quad (29)$$

that simplifies to

$$-a\eta - b + \phi b^2 - \phi b^2 p(t) = 0. \quad (30)$$

From (30), we obtain the optimal reinsured proportion as

$$p^*(t) = 1 - \left(\frac{a\eta + b}{\phi b^2}\right). \quad (31)$$

It is also dependent on time.

3.2 The Case of Correlated Shocks

In the case where the shocks correlate, equation (19) modifies to

$$dV = \frac{\partial V}{\partial t}dt + \frac{\partial V}{\partial w}\left\{\pi(t)\left[S(t)[\mu dt + \beta S^{\gamma}(t)dZ^{(1)}(t)]\right] + [W(t) - \pi(t)]\ kdt + \left(\theta - \eta p(t)\right)adt + \quad b(1 - p(t)\right\} + \frac{\partial^2 V}{\partial w^2}\{\,[\beta^2\pi^2(t)s^{2\gamma}(t) + 2\beta b\left(1 - p(t)\right)s^{2\gamma}(t)\pi(t)\rho\ + b^2\left(1 - p(t)\right)^2]dt\}. \quad (32)$$

Substituting (32) in (16) and simplifying, we get the modified Hamilton-Jacobi-Bellman (HJB) equation

$$V_t + \{(\mu S - K)\pi + kW + \left(\theta - \eta p(t)\right)a + b(1 - p(t))\}V_w + \left\{\beta^2\pi^2(t)s^{2\gamma}(t) + 2\beta b\left(1 - p(t)\right)s^{2\gamma}(t)\pi(t)\rho\ +\ b^2\left(1 - p(t)\right)^2]\right\}V_{ww} = 0. \quad (33)$$

Using (24) in (33) and simplifying yields the new HJB equation

$\frac{-1}{\phi}g' + \Big\{\{(\mu S - k)\pi + kW + (\theta - \eta p(t))a + b(1 - p(t))\} - \frac{\phi}{2}[\beta^2\pi^2(t)s^{2\gamma}(t) + 2\beta b(1 - p(t))s^{2\gamma}(t)\pi(t)\rho \ + b^2(1 - p(t))^2]\Big\}g = 0.$ (34)

Again, to obtain the optimal value $\pi^*(t)$ of $\pi(t)$ we differentiate (34) with respect to $\pi(t)$ to obtain

$(\mu S(t) - k) - \emptyset[\pi(t)\beta^2 s^{2(\gamma+1)}(t)] - \emptyset b\beta\rho(1 - p(t))s^{2(\gamma+1)}(t) = 0,$ (35)

From which we obtain the optimal investment strategy of the insurer to be

$\pi^*(t) = \frac{(\mu S(t) - k)}{\emptyset\beta^2 s^{2\gamma}(t)} + \frac{b\rho(p(t)-1)}{\beta}.$ (36)

Clearly, the insurer's optimal investment strategy when the shocks do not correlate can be recover for (36) if ρ equals zero, which is the implication of none correlation.

This insurer's optimal investment strategy in the risky asset (stock) is horizon dependent.

Also differentiating equation (35) with respect to $p(t)$, we obtain

$-a\eta - b + \emptyset b^2(1 - p(t))) + \rho\emptyset b\ \beta s^{2\gamma}(t) = 0.$ (37)

From (37), we obtain the insurer's optimal reinsured proportion as

$p^*(t) = 1 - \left(\frac{a\eta + b}{\phi b^2}\right) + \frac{\rho\ \beta s^{2\gamma}(t)}{b}.$ (38)

It is also dependent on time.

As in the case of the insurer's optimal investment strategy in the risky asset when the shocks do not correlate, which we recovered for (36) if ρ equals zero, the insurer's optimal reinsured proportion when the shocks do not correlate can be recovered for (38) under the same condition of ρ equals zero.

3.3. The Effect of Correlation of the Shocks.

In this segment we express the effect of correlation of the shocks thus

3.3.1. The case of the investment strategies in the risky asset

We compare (28), when the shocks do not correlate,

$\pi_{nc}{}^*(t) = \frac{(\mu S(t) - k)}{\emptyset\beta^2 s^{2\gamma}(t)}$

and (36) when the shocks do not correlate,

$\pi_c{}^*(t) = \frac{(\mu S(t) - k)}{\emptyset\beta^2 s^{2\gamma}(t)} + \frac{b\rho(p(t)-1)}{\beta}.$

It can be observed that

$\pi_c{}^*(t) = \pi_{nc}{}^*(t) + \frac{b\rho(p(t)-1)}{\beta}.$ (39)

The investment strategies differ by the fraction, $\frac{b\rho(p(t)-1)}{\beta}$.

If ρ is negative, that is

$\rho = -\vartheta,$ (40)

we have

$\pi_c{}^*(t) = \pi_{nc}{}^*(t) + \frac{\vartheta b(1-p(t))}{\beta}.$ (41)

Equation (41) implies the reverse of what is obtained in (40).

Also, if

$\rho = 1,$ (42)

we have

$\pi_c{}^*(t) = \pi_{nc}{}^*(t) - \frac{b(1-p(t))}{\beta},$ (43)

and when

$\rho = -1$ (44)

we get

$$\pi_c{}^*(t) = \pi_{nc}{}^*(t) + \frac{b(1-p(t))}{\beta}. \qquad (45)$$

3.3.2. The case of the reinsured proportion

We also compare the optimal reinsured proportion as in (31), when the shocks do not correlate,

$$p_{nc}{}^*(t) = 1 - (\frac{a\eta+b}{\phi b^2}).$$

and (38) when the shocks do correlate,

$$p_c{}^*(t) = 1 - \left(\frac{a\eta+b}{\phi b^2}\right) + \frac{\rho\,\beta S^{2(\gamma+1)}(t)}{b}.$$

It can be observed that

$$p_c{}^*(t) = p_{nc}{}^*(t) + \frac{\rho\,\beta s^{2\gamma}(t)}{b}. \qquad (46)$$

The optimal reinsured proportion differ by the fraction, $\frac{\rho\,\beta s^{2\gamma}(t)}{b}$.

Applying (40) in (46), gives

$$p_c{}^*(t) = p_{nc}{}^*(t) - \frac{\vartheta\,\beta s^{2\gamma}(t)}{b}, \qquad (47)$$

a reversal of (46).

Further, the application of (42) in (46) yields

$$p_c{}^*(t) = p_{nc}{}^*(t) + \frac{\beta s^{2\gamma}(t)}{b}, \qquad (48)$$

and using (44) in (46) gives

$$p_c{}^*(t) = p_{nc}{}^*(t) - \frac{\beta s^{2\gamma}(t)}{b}, \qquad (49)$$

which means that the reinsured proportion, when the shocks do not correlate is $\frac{\beta s^{2\gamma}(t)}{b}$greater than the insurer's reinsured proportion, when the shocks correlate.

3.4 Findings

1. The case of investment strategies: we found the investment strategies differ by the fraction, $\frac{b\rho(p(t)-1)}{\beta}$.
2. The case of the reinsured proportion: it found that the optimal reinsured proportion differ by the fraction, $\frac{\rho\,\beta s^{2\gamma}(t)}{b}$.

4. Conclusion

This work considered the impact of correlation of Brownian motions (shocks) on the optimal investment strategy and the reinsured proportion of the wealth an insurer had available for investment. It assumed that the insurer had the liberty to take proportional reinsurance and traded two assets in the market.

Ito's lemma was used to obtain the H-J-B equation from which optimal investment strategy and reinsured proportion were calculated. The optimal strategy was found to be a function of the price of the risky asset, and the square of the volatility of the risky asset when the Brownian motion did not correlate. When the Brownian motions were correlated, the optimal strategy was found to be a function of the price of the risky asset the square of the volatility of the risky asset and the reinsured proportion.

In the case of the reinsured proportion, when the shocks did not correlation, it was not horizon dependent. However when the shocks were correlated, it was found to be a function of the price and the volatility of the risky asset.

These impacts of the correlation of the Brownian motions should be considered when the insurer makes investment decisions.

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